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# SIMULATION OF THE ATMOSPHERIC ANNUAL ENERGY CYCLE [1]

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## ABSTRACT

A two level quasi-geostrophic model for zonally averaged conditions has been integrated for a period of a few years. The model is forced by Newtonian heating and has internal and surface friction. The interaction between the zonal flow and the eddies is simulated through the use of exchange coefficients for the transports of quasi-geostrophic potential vorticity and sensible heat.

The results of the integrations show that the model predicts a qualitatively correct annual variation of the zonal winds and the zonal temperature, although the predicted annual cycle has a too large amplitude compared with observations. The times of the maximum amounts of available potential and kinetic energy are well predicted as well as the typical time lag between the two quantities. The same statement holds for the generation of zonal available potential energy and the dissipation of zonal kinetic energy. The energy diagram obtained as an average for 1 yr of integration compares well with the corresponding diagram based on observations.

The major weakness of the model (i.e., the large annual variation of most quantities) is probably related to the simplicity of the thermal forcing.

## 1. INTRODUCTION

The annual variation of the energetics of the atmosphere has been the subject of three recent papers by one of the authors (Wiin-Nielsen 1967, 1969, 1970) the first of which deals with a diagnostic study of the various energy quantities based on data from a single year while the other two investigations attempt to simulate the behavior of the zonally averaged quantities (winds, temperatures, etc.) using a two level quasi-geostrophic zonally averaged model. The major problem in dealing with the zonally averaged quantities is to arrive at a model in which the equations are expressed entirely in the zonally averaged dependent variables of the model or, in other words, to simulate the interaction between the eddies and the zonal average in a realistic manner. This problem was not solved in the earlier investigations (Wiin-Nielsen 1969, 1970) because of the difficulties in expressing the momentum transport by the eddies in terms of the zonally averaged variables.

The closure problem is common to all studies in which the equations are written entirely in terms of the zonally averaged quantities. In these studies, it is necessary to parameterize the meridional transport of heat and momentum and express them in terms of the zonal wind and the zonally averaged temperature. Parameterization of the transport of heat, using an exchange coefficient, has been used by Adem (1962), Saltzman (1968), and Wiin-Nielsen (1970). It is much more difficult to parameterize the momentum transport. However, in a companion paper by the authors (1971), it was shown that it is possible to parameterize the momentum transport indirectly through the use of exchange coefficients for quasi-geostrophic potential vorticity and for heat. This approach to the problem was originally proposed by Green (1969). In the present study, it is only necessary to use the exchange coefficients for potential vorticity to integrate the problem in time, but the exchange coefficient for sensible heat enters when we want to compute quantities such as the mean zonal vertical velocity and the energy exchange between the zonally averaged quantities and the eddies.

The major objectives of this study are to formulate a two level quasi-geostrophic model for the zonally averaged wind and temperature, to integrate the model equations for a period of several years, and to see if it is possible to reproduce the major aspects of the annual variation of the general circulation. We are in particular interested in reproducing (1) the characteristic phase lag of approximately 1 mo between the maximum generation of available and the maximum destruction of kinetic energy through frictional dissipation and (2) the particular time of the year in which the amounts of energy attain their maxima. Observations show that this period is found in late January as shown by Wiin-Nielsen (1967). However, since the forcing of the atmospheric flow is of an extremely simple nature, as will be seen from the following sections, we do not expect that we will be able to reproduce the details of the zonal flow.

## 2. THE MODEL

We shall apply the standard two level quasi-geostrophic model in the study. The simplified vorticity equation is

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applied at the pressure levels 25 and 75 cb, designated by subscripts 1 and 3, respectively. The resulting equations are

$$\frac{\partial \zeta_1}{\partial t}+\mathbf{v}_1\cdot\nabla(f+\zeta_1)=\frac{f_0}{P}\,\omega_2+\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_2) \tag{1}$$

and

$$\frac{\partial \zeta_3}{\partial t}+\mathbf{v}_3\cdot\nabla(f+\zeta_3)=-\frac{f_0}{P}\,\omega_2+\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_4)-\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_2) \tag{2}$$

where $f$ is the Coriolis parameter; $f_0=f$ at 45°N; $P=50$ cb; $g$, the acceleration of gravity; $\zeta$, the relative vorticity; $\mathbf{v}$, the horizontal wind vector; $\omega_2$, the vertical velocity at 50 cb; $\boldsymbol{\tau}$, the stress vector; and $\mathbf{k}$, a vertical unit vector. Subscripts 2 and 4 refer to 50 and 100 cb, respectively.

Denoting the thermal stream function by $\psi_T$, defined as

$$\psi_T=\tfrac{1}{2}(\psi_1-\psi_3), \tag{3}$$

we can write the thermodynamic equation at level 2 as

$$\frac{\partial \psi_T}{\partial t}+\mathbf{v}_{1,3}\cdot\nabla\psi_T-\frac{\sigma P}{2f_0}\,\omega_2=\tfrac{1}{2}\,\frac{R}{c_p}\frac{1}{f_0}\,H_2 \tag{4}$$

where $H_2$ is the heating per unit mass and unit time; $\sigma=-(\alpha/\theta)(\partial\theta/\partial p)$ is the static stability; $\alpha$, the specific volume; $\theta$, the potential temperature; $p$, pressure; $R$, the gas constant; and $c_p$ the specific heat for constant pressure.

When $\omega_2$ is eliminated from eq (1) and (2) and using eq (4), we obtain

$$\frac{\partial Q_1}{\partial t}+\mathbf{v}_1\cdot\nabla Q_1=-\frac{R}{c_p}\frac{f_0}{\sigma P^2}\,H_2+\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_2) \tag{5}$$

and

$$\frac{\partial Q_3}{\partial t}+\mathbf{v}_3\cdot\nabla Q_3=+\frac{R}{c_p}\frac{f_0}{\sigma P^2}\,H_2+\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_4)-\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_2) \tag{6}$$

where

$$Q_1=f+\zeta_1-q^2\psi_T \tag{7}$$

and

$$Q_3=f+\zeta_3+q^2\psi_T \tag{8}$$

are the quasi-geostrophic potential vorticities at levels 1 and 3 and

$$q^2=\frac{2f_0^2}{\sigma P^2}\simeq 4\times 10^{-12}\ \mathrm{m}^{-2}. \tag{9}$$

Following the earlier papers (Wiin-Nielsen 1969, 1970), we approximate the diabatic heating, using a Newtonian form written as

$$H_2=-\gamma\,\frac{c_p}{R}\,2f_0(\psi_T-\psi_E) \tag{10}$$

where $\gamma$ is a constant found to be approximately $0.4\times10^{-6}$ $\mathrm{s}^{-1}$ and $\psi_E=(R/2f_0)\ T_E$ where $T_E$ is the climatological

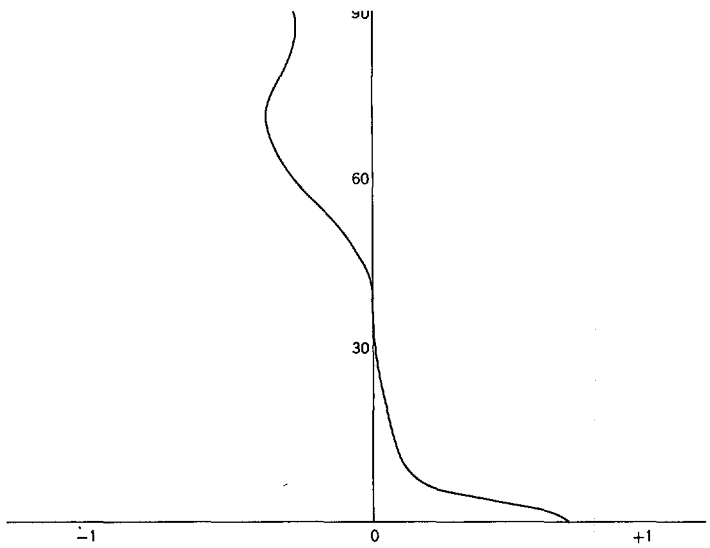


FIGURE 1.—Diabatic heating in January as function of latitude. Units are in the meter-ton-second system, approximately equal to degrees per day.

equilibrium temperature. Typical winter heating in January is given in figure 1. From the same investigations, we adopt the forms of the dissipative terms given by

$$\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_2)=-2A\zeta_T \tag{11}$$

and

$$\frac{g}{P}\,\mathbf{k}\cdot(\nabla\times\boldsymbol{\tau}_4)=-\epsilon\zeta_4 \tag{12}$$

where $A$ and $\epsilon$ were found to have the approximate values of $0.6\times10^{-6}\ \mathrm{s}^{-1}$ and $3\times10^{-6}\ \mathrm{s}^{-1}$, respectively. The $\zeta_T$ in eq (11) is defined by the relation

$$\zeta_T=\tfrac{1}{2}(\zeta_1-\zeta_3). \tag{13}$$

Defining furthermore

$$\zeta_*=\tfrac{1}{2}(\zeta_1+\zeta_3) \tag{14}$$

and

$$\zeta_4=\tfrac{1}{2}(3\zeta_3-\zeta_1), \tag{15}$$

we find by substitution of eq (10) and (12) in eq (5) and (6) that

$$\frac{\partial Q_1}{\partial t}+\mathbf{v}_1\cdot\nabla Q_1=\gamma q^2\,(\psi_T-\psi_E)-2A\zeta_T \tag{16}$$

and

$$\frac{\partial Q_3}{\partial t}+\mathbf{v}_3\cdot\nabla Q_3=-\gamma q^2\,(\psi_T-\psi_E)+2A\zeta_T-\epsilon\zeta_*+2\epsilon\zeta_T. \tag{17}$$

Equations (16) and (17) could be integrated if $\psi_E$ were specified as a function of time and the horizontal coordinates. However, since we are interested in reproducing certain aspects of the annual variation of the general circulation and therefore must integrate the equations over a period of at least 1 yr, it is desirable to search for the energy aspects in a further simplified model. It was therefore decided to form equations for the zonally averaged quantities from eq (16) and (17). Taking zonal averages of the equations and denoting this operation by a bar defined by

$$\overline{(\quad)}=\frac{1}{2\pi}\int_0^{2\pi}(\quad)\,d\lambda, \tag{18}$$

we obtain

$$\frac{\partial \overline{Q}_1}{\partial t}+\frac{1}{a\cos\phi}\frac{\partial(\overline{Q_1 v_1})\cos\phi}{\partial\phi}=\gamma q^2(\overline{\psi}_T-\overline{\psi}_E)-2A\overline{\zeta}_T \tag{19}$$

and

$$\frac{\partial \overline{Q}_3}{\partial t}+\frac{1}{a\cos\phi}\frac{\partial(\overline{Q_3 v_3})\cos\phi}{\partial\phi}=-\gamma q^2(\overline{\psi}_T-\overline{\psi}_E)+2A\overline{\zeta}_T-\epsilon\overline{\zeta}_*+2\epsilon\overline{\zeta}_T. \tag{20}$$

To express eq (19) and (20) entirely in zonally averaged quantities, one must at this point introduce an assumption relating the transport of quasi-geostrophic potential vorticity to the zonally averaged dependent variables of the model. As shown in eq (16), it is possible to do this by defining an exchange process by

$$(\overline{Qv})=-K\frac{\partial\overline{Q}}{a\partial\phi}. \tag{21}$$

When this is done in both eq (19) and (20), we obtain

$$\frac{\partial \overline{Q}_1}{\partial t}=\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(K_1\cos\phi\frac{\partial\overline{Q}_1}{\partial\phi}\right)+\gamma q^2(\overline{\psi}_T-\overline{\psi}_E)-2A\overline{\zeta}_T \tag{22}$$

and

$$\frac{\partial \overline{Q}_3}{\partial t}=\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(K_3\cos\phi\frac{\partial\overline{Q}_3}{\partial\phi}\right)-\gamma q^2(\overline{\psi}_T-\overline{\psi}_E)+2A\overline{\zeta}_T-\epsilon\overline{\zeta}_*+2\epsilon\overline{\zeta}_T. \tag{23}$$

The equilibrium stream function $\overline{\psi}_E=\overline{\psi}_E(\phi, t)$, necessary to integrate eq (22) and (23), was adopted from the study by Wiin-Nielsen (1970), where $\overline{\psi}_E$ was given as a function of latitude for each month. A simple interpolation formula was used to obtain $\overline{\psi}_E$ at 3-day intervals in the integration of eq (22) and (23). Knowing $\overline{\psi}_E$, one sees that the equations have the two dependent variables $\psi_*$ and $\overline{\psi}_T$. Assuming that these variables are known at a given time, we may use eq (22) and (23) to compute the values of $\overline{Q}_1$ and $\overline{Q}_3$ at the new time. The boundary conditions used in this study were that

$$\frac{\partial\overline{\zeta}_{1,3}}{\partial\phi}=0 \quad\text{and}\quad \frac{\partial\overline{\psi}_T}{\partial\phi}=0 \tag{24}$$

at $\phi=0$ and $\phi=\pi/2$.

It follows from eq (24) that

$$\frac{\partial\overline{Q}_{1,3}}{\partial\phi}=\frac{2\Omega}{a}\cos\phi \tag{25}$$

at $\phi=0$ and $\phi=\pi/2$.

When the new values of $\overline{Q}_1$ and $\overline{Q}_3$ have been obtained from eq (22) and (23), we can compute the new value of $\overline{\zeta}_*$ from the relations

$$\overline{\zeta}_*=\tfrac{1}{2}(\overline{Q}_1+\overline{Q}_3)-f. \tag{26}$$

The new value of $\overline{\psi}_T$ is obtained by solving the equation

$$\overline{\zeta}_T-q^2\overline{\psi}_T=\tfrac{1}{2}(\overline{Q}_1-\overline{Q}_3) \tag{27}$$

using the second boundary condition in eq (24). Equations (26) and (27) are obtained from eq (7) and (8) by addition and subtraction.

The initial conditions for the time integration were a constant value of the stream function $\overline{\psi}_T$ corresponding to a mean tropospheric temperature of 270°K and a vanishing value of $\overline{\zeta}_*$, or in other words a state of no motion. The equations were integrated in time, using an implicit scheme where the meridional derivatives were approximated by their time average, obtained from two time levels. The resulting scheme for a system with constant coefficients can be shown to be unconditionally stable. When variable coefficients are introduced as in the present study, numerical tests indicate that a time step of 6 hr results in good accuracy and stability. The grid size was chosen to be 2.5°.

The domain of integration is in this study limited to the Northern Hemisphere, using the boundary conditions (24) and (25). The possible importance of interhemispheric energy exchanges should be studied later in a model in which exchange coefficients are given from Pole to Pole. Present data limitations in the Southern Hemisphere do not permit a reliable estimate of the desired exchange coefficients.

## 3. ENERGY RELATIONS AND OTHER DERIVED QUANTITIES

The quantities available in each time step are $\overline{\psi}_T$ and $\overline{\zeta}_*$. Thus $\overline{\zeta}_T$ can be computed from $\overline{\psi}_T$, using the formula

$$\overline{\zeta}_T=-\frac{1}{a\cos\phi}\frac{\partial\overline{u}_T\cos\phi}{\partial\phi}=\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\overline{\psi}_T}{\partial\phi}\right). \tag{28}$$

The zonal winds $\overline{u}_*$ and $\overline{u}_T$ are computed from the relations

$$\overline{u}_*=-\frac{a}{\cos\phi}\int_0^\phi\overline{\zeta}_*\cos\phi\,d\phi \tag{29}$$

and

$$\overline{u}_T=-\frac{a}{\cos\phi}\int_0^\phi\overline{\zeta}_T\cos\phi\,d\phi. \tag{30}$$

Equation (29) is used to eliminate the calculation of $\overline{\psi}_*$, and eq (30) is employed to be consistent with eq (29).

The vertical velocity $\bar{\omega}_2$ can be calculated from the zonally averaged form of eq (4). Using the expression (10) for $H_2$, we obtain after taking the zonal average:

$$\frac{\partial\bar{\psi}_T}{\partial t}+\frac{1}{a\cos\phi}\frac{\partial(\overline{\psi_T v})\cos\phi}{\partial\phi}-\frac{\sigma P}{2f_0}\bar{\omega}_2=-\gamma(\bar{\psi}_T-\bar{\psi}_E). \quad (31)$$

As shown in eq (16), we may apply the approximation

$$(\overline{\psi_T v})=-K_H\frac{\partial\bar{\psi}_T}{a\partial\phi}. \quad (32)$$

Substitution of eq (32) in eq (31) results in the following equation for $\bar{\omega}_2$:

$$\bar{\omega}_2=\frac{2f_0}{\sigma P}\left(\frac{\partial\bar{\psi}_T}{\partial t}-\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(K_H\cos\phi\frac{\partial\bar{\psi}_T}{\partial\phi}\right)+\gamma(\bar{\psi}_T-\bar{\psi}_E)\right) \quad (33)$$

from which $\bar{\omega}_2$ may be computed at any desirable time.

The amounts of zonal available potential $(A_z)$ and zonal kinetic $(K_z)$ energy can be computed from $\bar{\psi}_T$ and the wind distributions. Subdividing the kinetic energy into the energy of the vertically averaged flow $K_{z*}$ and the energy in the deviation from this flow, the shear flow $(K_{zT})$, after Wiin-Nielsen (1962), we find referring also to Phillips (1956) that

$$A_z=\frac{P}{g}q^2\int_0^{\frac{\pi}{2}}\bar{\psi}^2_{T,d}\cos\phi\,d\phi, \quad (34)$$

$$K_{z*}=\frac{P}{g}\int_0^{\frac{\pi}{2}}\bar{u}^2_*\cos\phi\,d\phi, \quad (35)$$

and

$$K_{zT}=\frac{P}{g}\int_0^{\frac{\pi}{2}}\bar{u}^2_T\cos\phi\,d\phi \quad (36)$$

where the subscript $d$ in eq (34) indicates a departure from the area (in our case, meridional) mean.

The energy conversion from zonal available to zonal kinetic energy is

$$C(A_z,K_z)=-\frac{2f_0}{g}\int_0^{\frac{\pi}{2}}\bar{\psi}_T\,\bar{\omega}_2\cos\phi\,d\phi, \quad (37)$$

while the conversion from zonal to eddy available potential energy can be written in the form

$$C(A_z,A_E)=-\frac{2P}{g}q_0^2\int_0^{\frac{\pi}{2}}\bar{K}_H\bar{\psi}_T\bar{\zeta}_T\cos\phi\,d\phi. \quad (38)$$

While the generation of zonal available potential energy can be computed directly from the usual formula, it was found convenient to use

$$G(A_z)=\frac{dA_z}{dt}+C(A_z,A_E)+C(A_z,K_z) \quad (39)$$

for the calculation of $G(A_z)$.

The most convenient formula for calculation of the energy conversion from eddy to zonal kinetic energy was found to be

$$C(K_E,K_z)=-\frac{P}{g}\int_0^{\frac{\pi}{2}}\left(K_1\bar{u}_1\frac{\partial\bar{Q}_1}{a\partial\phi}+K_3\bar{u}_3\frac{\partial\bar{Q}_3}{a\partial\phi}\right)\cos\phi\,d\phi+C(A_z,A_E) \quad (40)$$

which follows from the original formula stated by Phillips (1956) by a substitution of the formula for the convergence of the momentum transport as derived by the authors (Wiin-Nielsen and Sela 1971). The dissipation of kinetic energy was finally computed as a residue from

$$D(K_z)=C(A_z,K_z)+C(K_E,K_z)-\frac{dK_z}{dt}. \quad (41)$$

Although the expression (41) was used in calculating $D(K_z)$, it is instructive to inspect the formula for this quantity. We find that

$$D(K_z)=\frac{2P}{g}\int_0^{\frac{\pi}{2}}(\epsilon\bar{u}_3\bar{u}_4+\tfrac{1}{2}A(\bar{u}_1-\bar{u}_3)^2)\cos\phi\,d\phi. \quad (42)$$

The last part of the integrand in eq (42) measures the dissipation due to the internal stresses and is always positive. The first part of the integrand measures the dissipation due to the surface stresses. It will give a positive contribution if $\bar{u}_3$ and $\bar{u}_4$ are positively correlated. Since $\bar{u}_4$ is obtained by a linear downward extrapolation from the values of $\bar{u}_1$ and $\bar{u}_3$, there is no assurance that the contribution will always be positive. If the predicted vertical wind shear is too large, we may obtain surface easterlies while there are westerlies at level 3. As a consequence, it may turn out that $D(K_z)$ becomes negative especially because $\epsilon$ is large compared to $A$. This unphysical behavior of the model is naturally due to the rather arbitrary, but conventional procedure to obtain $\bar{u}_4$ by extrapolation as indicated in eq (15) and could have been avoided if we had made $\bar{u}_4$ proportional to $\bar{u}_3$ by some factor as, for example, done by Charney (1959).

## 4. RESULTS

Using the procedure described at the end of section 2, we integrated the model equations for a period of over 2.5 yr. The numerical values of $\gamma$, $\epsilon$, $A$, and $q$ were given in section 2 [see eq (9–12)]. The exchange coefficients $K_1(\phi)$ and $K_3(\phi)$ were those derived in the diagnostic study by the authors (Wiin-Nielsen and Sela 1971), and they are also shown in figure 2 of this paper. As mentioned in section 2, we used the basic time step of 6 hr; but the zonal winds, temperatures, and energy quantities were only calculated every third day. The purpose of the remaining part of this section is to discuss some of the results. Since the initial distribution corresponds to a state of rest, it takes some time to generate the zonal

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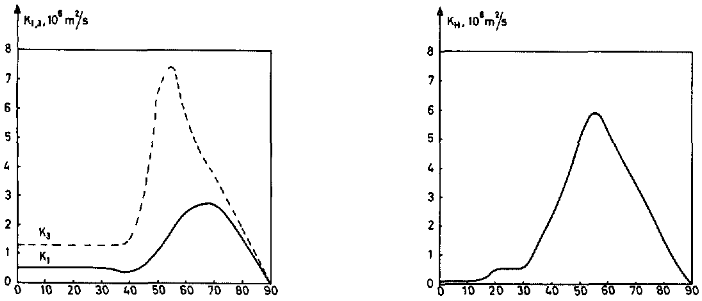


FIGURE 2.—(A) exchange coefficients for the transport of quasi-geostrophic potential vorticity at level 1 and level 3, respectively, as a function of latitude and (B) exchange coefficient for the transport of sensible heat as a function of latitude.

wind fields and to modify the initial temperature field. The initial date is January 1. It is therefore seen that the first year of integration can not be used to study the problem in which we are interested (i.e., the annual variation of the general circulation). The data discussed in the following are therefore those corresponding to the second year of integration. The integration for a part of the third year shows that the values obtained here are almost identical to the second year.

Before presenting the computed zonal wind distributions through the year, it should be mentioned that the computed wind speeds have been corrected in such a way that the annual mean value of the meridional average of the vertically averaged zonal wind remains constant. The reason for this is as follows. The zonal averages of the first equation of motion at levels 1 and 3 are

$$\frac{\partial \bar{u}_1}{\partial t}+\frac{1}{a\cos^2\phi}\frac{\partial(\overline{u_1v_1})\cos^2\phi}{\partial\phi}=f\bar{v}_1+\frac{g}{P}\bar{\tau}_2 \tag{43}$$

$$\frac{\partial \bar{u}_3}{\partial t}+\frac{1}{a\cos^2\phi}\frac{\partial(\overline{u_3v_3})\cos^2\phi}{\partial\phi}=f\bar{v}_3+\frac{g}{P}\bar{\tau}_4-\frac{g}{P}\bar{\tau}_2. \tag{44}$$

Since $\bar{v}_1=-\bar{v}_3$, we obtain by adding eq (43) and (44) and dividing by 2 that

$$\frac{\partial \bar{u}_*}{\partial t}+\frac{1}{2}\frac{1}{a\cos^2\phi}\left[\frac{\partial(\overline{u_1v_1})\cos^2\phi}{\partial\phi}+\frac{\partial(\overline{u_3v_3})\cos^2\phi}{\partial\phi}\right]=\frac{g}{2P}\bar{\tau}_4. \tag{45}$$

Equation (45) is integrated from Equator to Pole after it has been multiplied by $a\cos^2\phi$. We obtain, using the fact that there is no momentum flux across the Equator in our model,

$$\frac{\partial}{\partial t}\left(\int_0^{\frac{\pi}{2}}\bar{u}_*\,a\cos^2\phi\,d\phi\right)=a\frac{g}{2P}\int_0^{\frac{\pi}{2}}\bar{\tau}_4\cos^2\phi\,d\phi. \tag{46}$$

In the average over 1 yr, we must expect the left side to vanish. It is therefore seen that the integral

$$\frac{1}{T}\int_0^T\int_0^{\frac{\pi}{2}}\bar{\bar{\tau}}_4\cos^2\phi\,d\phi=0. \tag{47}$$

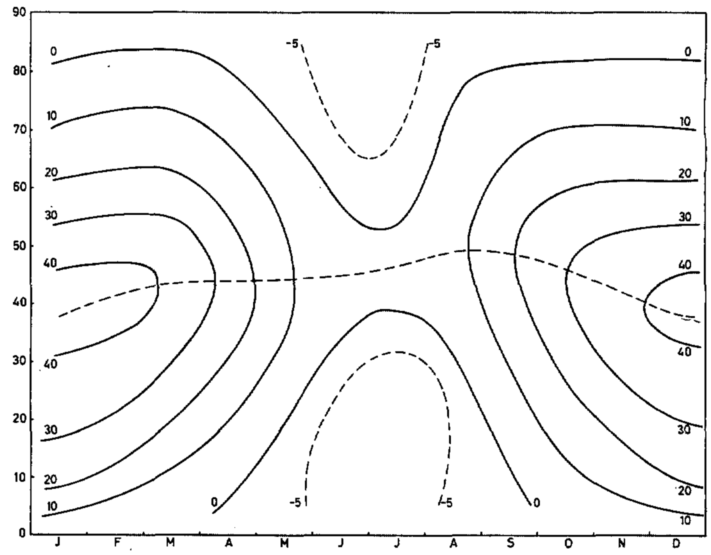


FIGURE 3.—Zonally averaged wind at level 1 (25 cb) as a function of time and latitude; units, m/s. Positive values indicate flow from the west.

The $\bar{\bar{\tau}}_4$ is in our model proportional to $\bar{u}_4$. It follows that the integral

$$\int_0^{\frac{\pi}{2}}\bar{\bar{u}}_4\cos^2\phi\,d\phi=0. \tag{48}$$

where $\bar{\bar{u}}_4$ is the annual mean value of the computed zonal wind speeds at level 4. The integral (48) was computed from the results, and the resulting value subtracted from $\bar{u}_1$, $\bar{u}_3$, and $\bar{u}_4$.

The zonal winds are plotted as a function of time of the year and latitude in figures 3, 4, and 5 showing the winds at the levels 1, 3, and 4, respectively. The maximum winds at level 1 (25 cb) appear in the middle latitudes (40°N) with speeds just less than 50 m/s. We observe that the maximum winds are found somewhat farther to the north (10° latitude displacement) during summer, in qualitative agreement with observations. In comparison with observations (see Lorenz 1967), it is seen that the winds are somewhat too strong during winter at level 1, but too weak during the summer. This general feature of the model (i.e., a too marked annual variation) will be found in most of the quantities in the model. Easterlies appear in the low latitudes at level 1 around May, obtain their maximum strength in July (>5 m/s), and disappear again in September–October. This behavior of the model is also in qualitative agreement with observations (Lorenz 1967).

The maximum winds at level 3 (75 cb) are found at 50°N, approximately, with some displacement to the north in the summertime. As at the upper level, we have too strong winds during winter and too weak winds during summer as compared to observations. The easterlies in the low and high latitudes occur at approximately the same time as at the upper level, but they are weaker, in agreement with observations. The zonal winds at level 4

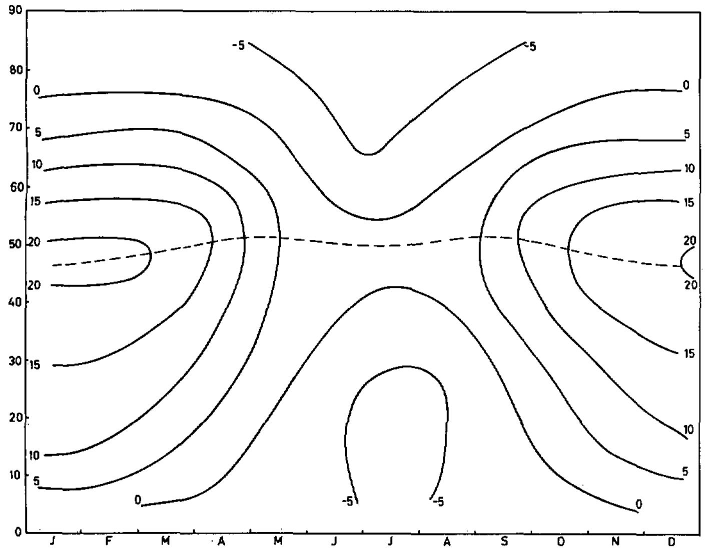

FIGURE 4.—Zonally averaged wind at level 3 (75 cb) as a function of time and latitude; units, m/s.

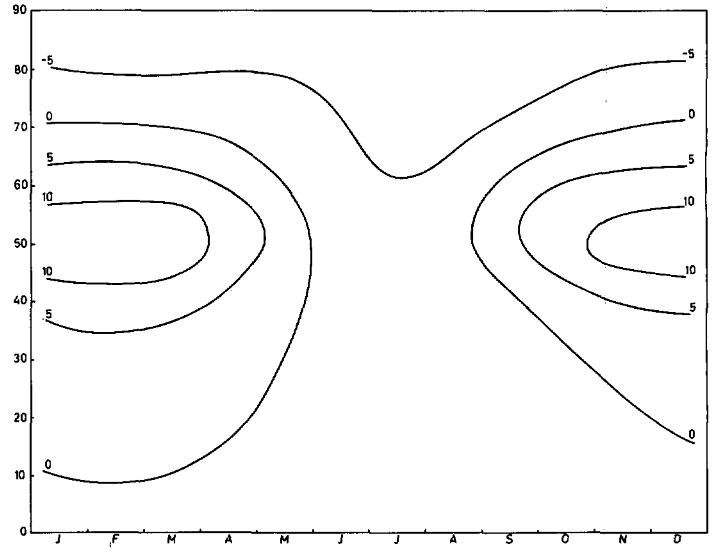

FIGURE 5.—Zonally averaged wind at level 4 (100 cb) as a function of time and latitude; units, m/s.

(100 cb), obtained by a linear downward extrapolation, show essentially the same features as figure 3, but it should be mentioned that one obtains easterlies at all latitudes during August.

The more marked annual variation found in the model as compared to the observed state of the atmosphere could perhaps be traced to the use of the adopted exchange coefficients. However, numerical experiments in which $K_1$ and $K_3$ were varied through the year, simulating the observed variation (16), indicate that such a reduction of the meridional transport of potential vorticity during the summer months and an increase during the winter months is insufficient for correcting the meridional temperature gradient. The major reason for the discrepancy between the model and the observed state of the atmosphere is most likely the simplicity of the heating, which in our case is of a Newtonian form, where the equilibrium temperature is determined from a very simple radiational theory as in the earlier paper by Wiin-Nielsen (1970). Further experiments with models of the kind used here must, in our opinion, be done with a heating function that is more realistic than the one used in this study.

The need for an improved heating function can also be seen from an analysis of the predicted temperature at level 2 shown in figure 6 as a function of time and latitude for the year. The temperature distribution is characterized by a maximum temperature gradient in the middle latitudes during winter. An inspection of the data shows that the absolute greatest temperature gradient that can be found in the field is 11°C per 10° latitude at 40°N during January and February. This value corresponds to a vertical wind shear of 4.5 $\text{m}\cdot\text{s}^{-1}\cdot\text{km}^{-1}$ computed from the geostrophic thermal wind equation. Such a value is in very good agreement with observations. The main discrepancies between the computed temperature field and

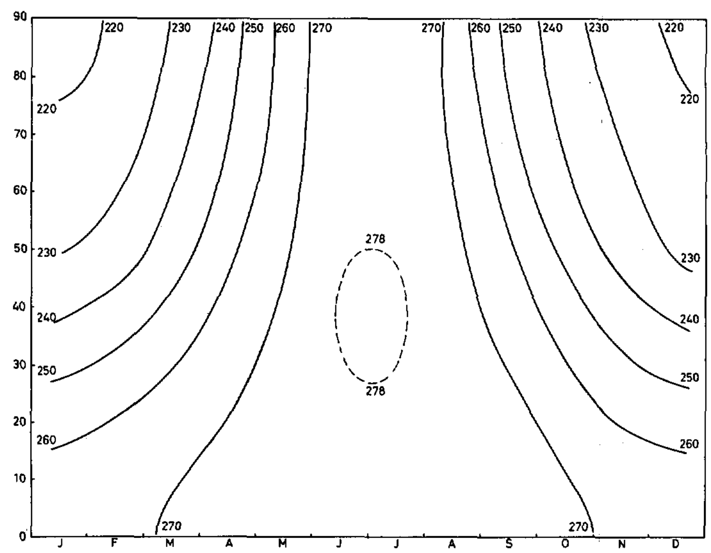

FIGURE 6.—Zonally averaged temperature at level 2 (50 cb) as a function of time and latitude; units, °K.

observed meridional cross sections (see Lorenz 1967 and Palmén and Newton 1969) are that the temperatures are too low in winter in the high latitudes and too high in summer at the same locations and that there is almost no meridional temperature gradient during July and August in the model results. The results for the temperature distribution are naturally in agreement with the predicted wind distributions discussed above.

The main reason for the investigation described here was to see if it was possible to reproduce the characteristic phase differences between energy quantities found in observational studies by Wiin-Nielsen (1967) and Kung and Soong (1969). The main conclusion from the diagnostic study was that the maximum available potential

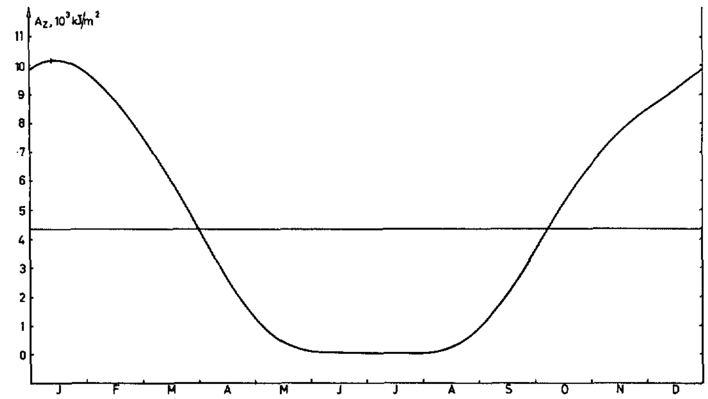


FIGURE 7.—Zonal available potential energy as a function of time; units, $10^3$ kJ/m².

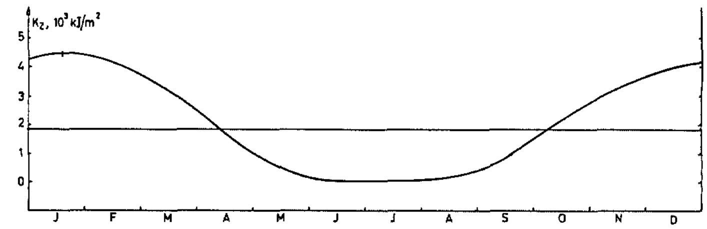


FIGURE 8.—Zonal kinetic energy as a function of time; units, $10^3$ kJ/m².

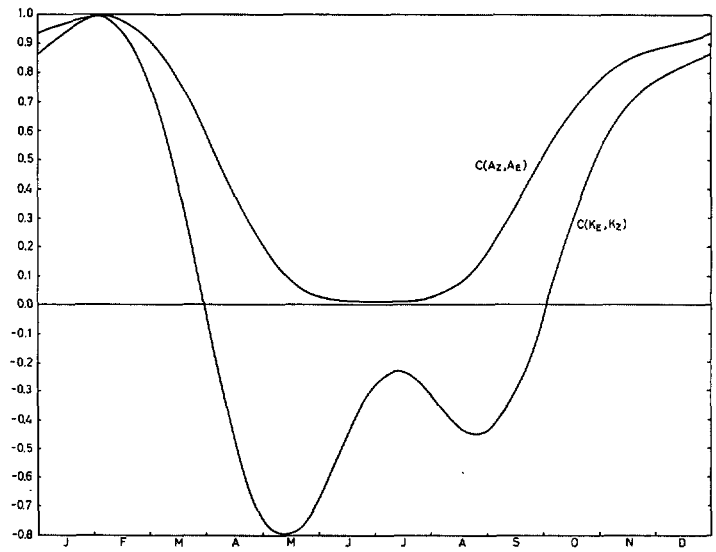


FIGURE 9.—Energy conversion between zonal and eddy available potential energy (upper curve) and between eddy and zonal kinetic energy (lower curve) as a function of time. Each quantity is normalized by its maximum value during the year.

energy occurs a few days before the maximum dissipation of kinetic energy. These results are based on the first Fourier component of the annual variation and are obtained from only 1 yr of data. In the following, we shall discuss the results obtained from the model.

Figures 7 and 8 show the available potential and the kinetic energy in the zonal flow as a function of time for the second year of integration. It is seen that both quantities attain their maximum in the middle of January, in excellent agreement with the observational study (Wiin-Nielsen 1967). The time of maximum energy is marked on each curve. This time is the 13th day of the year for $A_z$ and the 19th day for $K_z$, or a phase difference of 6 days, which also is in good agreement with observations. As could be expected from the temperature and wind distributions given in figures 3–6, we find again a too large annual variation with extremely small amounts of energy during the summer.

All curves showing variations with time were plotted directly from output taken every third day.

An investigation of the mean meridional circulation reveals the same sensitivity to the heating as experienced by the winds and the temperature field. A typical cross section for wintertime conditions shows rising motion from the Equator to 55°N with sinking motion from there to the Pole. There is a definite minimum at 30°N, indicating a tendency for a three-cell circulation. No vertical velocities exceed 3.5 mm/s, in good agreement with observations. Summer conditions are not satisfactory, resulting from the crudeness of the heating, although they remain extremely small.

To make the comparison of the generation, the conversions, and the dissipation easier, expecially with respect to the time differences, we have normalized each of these quantities, using the maximum value for the year as the normalizing factor. Figure 9 shows the annual variation of $C(A_z, A_E)$ (upper curve) and $C(K_E, K_z)$ (lower curve). It is easily seen from eq (38) that $C(A_z, A_E)$ is a positive quantity at each instant, while $C(K_E, K_z)$ may have both positive and negative values. The two energy conversions attain their maximum at about the same time (early February). When compared to the observational study by Wiin–Nielsen (1967), we find good agreement for $C(A_z, A_E)$ while there is a marked disagreement for $C(K_E, K_z)$. However, the observational study was made for the year 1963 which was most unusual in the early part of the year during which time $C(K_E, K_z)$ is negative as shown by Wiin–Nielsen et al. (1964). We notice furthermore that $C(K_E, K_z)$ in the model depends crucially on the parameterization of the momentum transport through the exchange coefficients for potential vorticity and heat. Figure 9 shows that the parameterization operates in such a way that $C(K_E, K_z)$ is positive in the average and during the major part of the year, although it turns to small negative values during the summer. This means that the momentum transport through the major part of the year is transported from regions of low zonal wind speeds to regions of high zonal wind speeds as is observed on the average in the atmosphere (e.g., Starr 1966).

Figure 10 shows the annual variation of the generation of zonal available potential energy and the dissipation of zonal kinetic energy as a function of time. We notice first

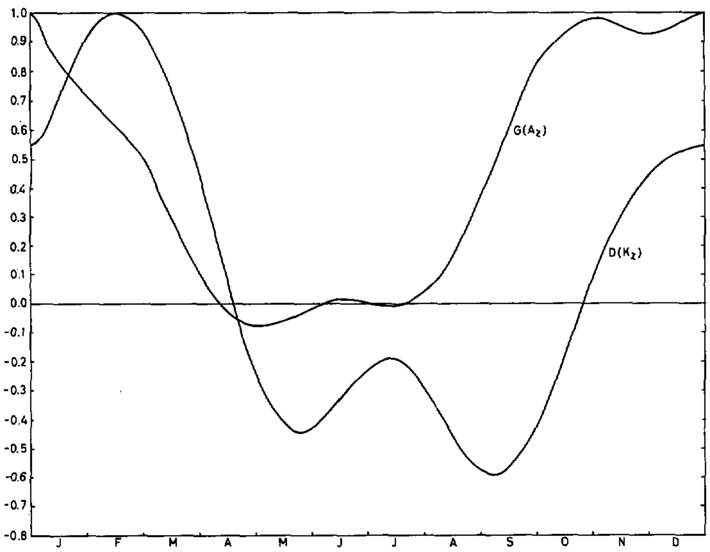


FIGURE 10.—Generation of zonal available potential energy $G(A_z)$ and the dissipation of zonal kinetic energy $D(K_z)$ as a function of time. Normalization is the same as in figure 9.

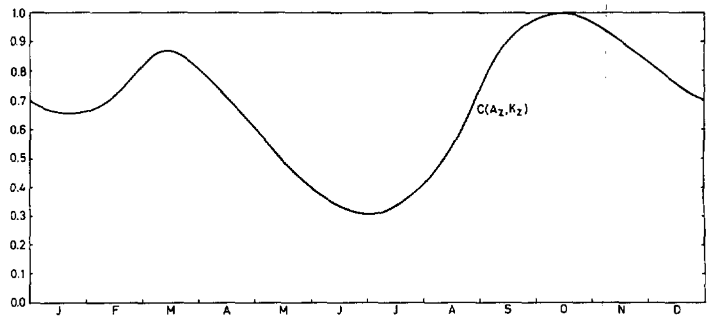


FIGURE 11.—Energy conversion from zonal available potential to zonal kinetic energy as a function of time. Normalization is the same as in figure 9.

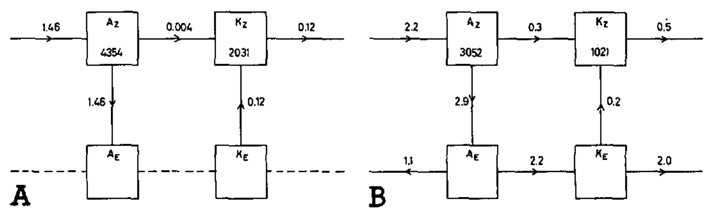


FIGURE 12.—(A) energy diagram for the model showing averages for the second year of integration and (B) energy diagram based on observations.

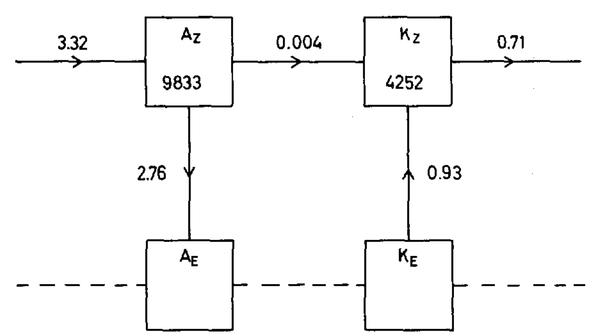


FIGURE 13.—Energy diagram for the model corresponding to the time where $G(A_z)$ is a maximum.

of all that the model has reproduced the observed phase lag between the two quantities. The maximum for $G(A_z)$ during winter is rather broad and centers around the end of November, while the maximum for $D(K_z)$ is found in the middle of February. An evaluation of the first Fourier component from the time series displayed in figure 10 shows that the phase angle for $G(A_z)$ is $-25$ days, while the phase angle for $D(K_z)$ is 22 days. The phase difference is therefore approximately 1.5 mo, which shows that this relative simple model for the zonal flow of the atmosphere is capable of producing results which are in good agreement with observational studies with respect to the absolute and relative positions of the major energy components during the annual cycle.

From figure 10, we notice that $D(K_z)$ is negative during the summer months. This unrealistic feature is due to the linear extrapolation procedure used to obtain the surface winds as discussed at the end of section 3, see eq (42). The last energy conversion that can be computed from the model is $C(A_z, K_z)$ presented in figure 11. It shows that the mean meridional circulation is operating in a thermally directed way on the average throughout the year with the maximum intensity between October and March. The minimum intensity is found during the summer. This picture is in good agreement with the few existing observational studies of the mean meridional circulation and the energy conversion $C(A_z, K_z)$.

The curves presented in figures 9–11 were normalized with respect to the maximum value during the year of the quantity. It is naturally also of interest to investigate the absolute values of the energy quantities. We shall first consider the annual mean values obtained by averaging the time series for the second year of integration. These values are presented in figure 12, using the well-known diagram originally introduced by Phillips (1956). Also included in the same figure is our reproduction of an energy diagram obtained from the data given by Wiin-Nielsen (1968). By comparison of the two diagrams, we observe that the model behaves qualitatively correctly in the sense that the energy flow is the same in the two diagrams. It is furthermore seen that there is very good agreement in the magnitudes of the major generations, conversions, and dissipations. The only difference is the conversion $C(A_z,K_z)$, which is an order of magnitude smaller in the model than in the data studies; but it must be stressed that the magnitude of the conversion is rather uncertain in observational studies due to the difficulty in assessing the mean meridional circulation. From the amounts of energy, it is seen that they are somewhat larger in the model than in the atmosphere, in agreement with the remarks made in connection with the distributions of temperatures and winds.

It may also be of interest to consider an instantaneous picture of the energetics of the model. As an example, we have chosen the time where the generation of zonal available potential energy is a maximum (approx. November 1 in the model). The resulting energy diagram is shown in figure 13. In this case, we do not obtain a balance for each energy reservoir. The reason is naturally that this is a time where both $A_z$ and $K_z$ are increasing as seen from figures 6 and 7.

## 5. CONCLUDING REMARKS

It has been demonstrated that the integration of a two level quasi-geostrophic model for the zonal flow is capable of reproducing the major aspects of the annual variation of the zonal flow, the zonally averaged temperature, and the various components of atmospheric energetics. The interaction between the eddies and the zonal flow is in this model simulated by exchange processes, using exchange coefficients derived from atmospheric data. The model has internal and surface friction, and the forcing of the flow is through Newtonian heating.

The results of the integration over a period of about 2.5 yr show that the model behaves best in predicting the time of the maximum for the amounts of energy and the generation, conversion and dissipation, and in particular the positions of these maxima relative to each other. The main weakness of the model is that it predicts an annual variation that is too large for almost all quantities. This is especially apparent in the annual variation of the zonal available potential and zonal kinetic energies. We have been able (based on numerical experiments) to trace this behavior to the simplicity of the Newtonian heating and, in particular, to the specification of the equilibrium temperature field determined from a simple radiation equilibrium calculation. It is naturally known that other components of the heat budget are very important such as the exchange of heat between the atmosphere and the underlying surface and perhaps also the heat of condensation. The present paper should therefore be considered as a feasibility study of a model of the present kind. It is our opinion that the results are sufficiently encouraging to warrant further use of models for the zonal flow in which the interaction between the eddies and the zonally averaged quantities is parameterized in some way.

It is undoubtedly possible to improve upon the heating used in this study. Although the heat exchange with the earth's surface may be relatively easy to include, even though the distribution of continents and oceans create a problem, it is much more difficult to model the heat of condensation. However, it is apparent from this study that the processes must be modeled in some fashion if we want a closer agreement with observations than found here.

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